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THE MATHEMATICS TEACHER

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THE CURRICULUM—PRESENT TENDENCIES, FUTURE POSSIBILITIES.

BY J. T. RORER.

Introduction.

There is probably no better evidence of the increased interest in popular education than the present tendency to carefully scrutinize all parts of our educational machinery to note the defects and their remedy, to suggest improvements, and how to introduce them. The attitude of mind that conforms to the injunction, "Prove all things," is most healthful and should bring about better results. As teachers of mathematics we have nothing to fear from this critical spirit. We have proved all things from ancient times and our triumphal Q.E.D. has appeared over and over again. Shall we then not expect our own attitude of demanding proof to be a natural and reasonable one for others?

Mathematics has formed a part in all systems of education in all times, in varying amounts to be sure. In the days of our grandfathers the common school education was reading, writing and arithmetic. Modern development has brought about the introduction of many other studies. So that the modern course of study is exceedingly varied and complex, and we find our educational system from the bottom to the top criticized as being superficial and giving a smattering of too many subjects with a scholarly command of none. I believe we can all agree

upon this statement at least,—that the high school pupil of to-day faces too many studies at one time and that this dissipates his energy.

There has come into being within the last few years a new censor of education, The Carnegie Foundation for the Advancement of Teaching. The scholarship, the sincerity, the independent fearlessness of the management of this foundation, makes its utterances worthy of attention of all teachers. In the fifth annual report, the quality of high school and college training is considered from three points of view: (1) The testimony of the college entrance tests; (2) the testimony of employers of high school and college graduates; (3) the testimony of Oxford tutors. I will briefly give President Pritchett's conclusions from these three kinds of testimony.

Concerning college entrance tests, President Pritchett says: "A majority of the students entering college by examinations failed to fulfil the tests set up by these examinations. . . . In 1910, the College Entrance Examination Board examined about two thousand students; of this number about one half were not able to be rated as high as sixty per cent."

President Pritchett comments upon the testimony of business men: "While, therefore, it is impossible to summarize in a brief statement all that is contained in the criticism of business men concerning the performance of high school and college graduates, it may justly be said that business men would much prefer to receive, whether from high school or from college, the young man who has learned to do something well, and who has acquired the mental discipline which has come from this experience and who is ready to receive directions and follow them, than to take into their business either from high school or college, the ordinary representative, who comes with a little knowledge of many things, with no taste for the grind of business or of scholarship, and who lacks that grasp of his own mind which can come only from having acquired a mastery of some one thing."

In summarizing the testimony of the Oxford tutors, President Pritchett says, in part: "These observations . . . show also most clearly that in the majority of cases the (Rhodes scholarship) student finds difficulty in doing his work, arising out of

the superficiality and the diffuseness of his previous training in the American college, and the failure of this training to give him intellectual power. It is the characteristic weakness of the whole American life. The American has not yet got out from the spell of the pioneer days. He is alert, resourceful, but superficial. The pioneer stage itself has passed. Alertness and resourcefulness can no longer take the place of thorough training and careful preparation. To-day the great opportunity of the secondary school and of the college is to cure this weakness, not to minister to it."

If necessary, much corroborative testimony might be presented to establish the fact that our present curricula, as applied to the individual student, are absurdly diffuse and that this diffuseness has had for many years a weakening influence on school and college work. Teachers everywhere have noted this unfortunate condition; it is only recently, however, that a revolt has begun against this wrong.

COLLEGE ENTRANCE.

College entrance requirements have covered too many subjects, and the school courses have included even a wider range. In recent years the secondary schools have responded to the popular demand for courses that applied closely to the present and the future needs of the pupils along vocational lines, manual training, bookkeeping, stenography, and typewriting, agriculture, sewing, cooking, etc. We can give these manual subjects credit for breaking the fetters of tradition and opening the way of reform. Clearly, if a considerable part of the five or six hours of the high school day is given for such vocational work, the remaining time is insufficient for the ground prescribed by the colleges. In the east, practically no credit has been given for such vocational work, hence pupils taking such courses have been unfairly handicapped when asking for college admission.

During the past year, several official reports have been made against these and other unhappy conditions which I have not taken time to mention. Among these reports let me note the following: Resolutions of the Secondary Department of the National Educational Association; Resolution of the Depart-

ment of Manual Training and Art Education, N. E. A.; Resolutions of Department of Business Education, N. E. A.; Report of the Committee of the New York High School Teachers' Association; Report of the Committee on College Entrance Requirements of the Federation of Teachers of the Mathematical and the Natural Sciences; Report of the Boston Head Masters' Association.

The demands of these reports and resolutions are chiefly the reduction of the number of required subjects, and the recognition of a wider range of subjects, especially of the vocational subjects.

Some response has already been made. Harvard is starting an influence in the right direction in her new entrance requirements which specify examinations in four subjects as follows:

- (A) English.
- (B) Latin, or, for candidates for the degree of S.B.,
French or German.
- (C) Mathematics, or physics, or chemistry.
- (D) Any subject (not already selected under (B) or (C))
from the following list:
 - Greek, History, Physics,
 - French, Mathematics, Chemistry.
 - German,

In addition to facing these four examinations the candidate must produce a statement showing: "(a) A four years' course of study; (b) that his course has been concerned chiefly with languages, sciences, mathematics, and history, no one of which has been omitted; (c) that two of the studies of high school program have been pursued beyond their elementary stages, *i. e.*, to the stage required by the present advanced examinations of Harvard College or the equivalent examination of the College Entrance Examination Board."

These requirements make it possible that no more than five subjects occupy the attention of the students at one time. This does not mean that fewer subjects must be taught to the individual and necessarily implies that each boy and girl shall select under guidance those subjects which seem to be best adapted to his present and future needs.

PEDAGOGIC VALUES.

The agitation for a change in the curriculum leads to an examination of subjects and methods of teaching which shall reveal their pedagogical values.

In recent years our teachers of mathematics have divided into two camps: first, those who claim that mathematics is only justified in the curriculum for its practical application; second those who claim that its cultural value, its power to train the reasoning faculties, is its greatest argument for existence in the curriculum.

If one's lot is to teach a group of specialized students, a group of engineers, for instance, he may sound the practical application idea with considerable force. If, however, it is your lot to believe in algebra, geometry, trigonometry, etc., *for the multitude*, how can you claim that your average student will ever in his mature years square a binomial or a hypotenuse, or obey the law of sines? Accordingly, this latter group of teachers who are aware of actual conditions and who are honestly presenting mathematics without extravagant claims for its daily application, must examine carefully the faith that is in them. Is power cultivated in mathematical studies transferable to the non-mathematical problems of life? In what way will algebra and geometry make a better salesperson, physician, nurse or journalist?

The present trend of pedagogy is to discover tests of efficiency in producing thought power. Our opponents claim that mathematical training at best but develops certain brain cells which, when they respond, produce the successful mathematical thinker; that these cells are narrow specialists and their training is as useless for other kinds of thinking as is the use of chest weights to produce technique on the piano. Dr. George E. Dawson, in *Popular Science Monthly* (September, 1910) says: "The intellectual culture derived through standardized branches of education, as mathematics and Latin, for example, instead of having a general mental economy for the innumerable young men and women who study them, in reality becomes parasitic in the nervous and mental life, and this is a cause of wasted energy and possibly, of disease."

Dr. Dawson then refers to the over-trained athlete typical of

our teams whose subsequent life is actually shortened by a strained heart or weakened lungs and claims that "his vitality is reduced through parasitic muscular culture."

Such an argument takes into account but few cases of misdirected and narrow specialization. No intelligent physical director will to-day endorse such training. It is common observation that where *one* suffers from over training hundreds, perhaps *thousands* suffer from insufficient training. To compare ordinary school and college courses in languages and mathematics to such physical over-exertion and straining is manifestly absurd. We must also dissent from the opinion that mathematical culture and power are not transferable to other phases of mental action. The weight of opinion, past and present, is strongly on our side. Mathematicians are sometimes cartooned as freaks—out of place in the world's work. Such is not the teaching of history; nor, as a class, are the members of the American Mathematical Society, for instance, in any way more abnormal than the members of any other group of educated people.

THE TESTS OF MODERN PSYCHOLOGY.

It may seem presumptuous for one untrained in modern psychology to criticize the opinions and evidence of the masters of that science. Nevertheless, I wish to make a few comments on some of the statements of the quantitative psychologists, particularly on the topic of correlation.

Correlation is defined "as a tendency of two or more traits or capacities to vary together."* It is what we might term *functionality* of ability, capacity, or trait.

"A correlation is indifferent or *zero* if the existence or variation of one trait is totally unrelated to that of the other. In *perfect positive* correlation (direct linear variation) the correlation is unity or 1.00. . . . In actual psychological investigation, at least when functional correspondences are under investigation, we have commonly to deal with some intermediate degree of correspondence, and *r*, the correlation coefficient, assumes therefore the form of a decimal lying between 0 and

* Whipple, "Mental and Physical Tests," p. 28.

+ 1.00 for positive, and 0 and — 1.00 for negative correspondence.”*

The formulas used in the computation of r by the product-moments method of Pearson are:

$$r = \frac{\sum xy}{n\sigma_1\sigma_2}$$

in which x = the residual of a single observation of the first kind from the mean of all the observations of the first kind;

y = the similar residual from observations of the second kind;

n = the number of individuals observed;

$$\sigma_1 = \sqrt{\frac{\sum x^2}{n}}, \quad \sigma_2 = \sqrt{\frac{\sum y^2}{n}},$$

where $\sum x^2$ = the sum of the squares of the residuals of the first kind; and $\sum y^2$ = the sum of the squares of the second kind.

In order to estimate fairly the results of correlation studies pertaining to mathematics, a few correlation results are given in other subjects.

Test of 50 boys (Whipple), strength of grip comparing right and left hands, $r = .93 \pm .02$.

Correlation of stature of father and son (Pearson), $r = .396$.

Stature of brother and brother (Pearson), $r = .391$.

The problem of the correlation between efficiency in mathematics and in other subjects is commanding the attention of several quantitative psychologists. Thus far, the results of these investigations are by no means in agreement. Rietz and Shade† have found a decided positive correlation between efficiency in mathematics and efficiency in the natural sciences and in languages.

Mathematics and foreign languages, $r = .476$.

Mathematics and natural science, $r = .440$.

Below, in column 1 are the results from study of about 500 pupils in sixteen high schools by W. P. Burris; in column 2 are the results of 232 New York Regents' Examinations, but not

* Whipple, "Mental and Physical Tests," pp. 28 and 29.

† *University of Illinois Bulletin*, Vol. VI., No. 10.

more than 108 cases in any one computation, by Brinckerhoff, Morris and Thorndike.*

Correlation Between	1.	2.
Mathematics and English	.39	.09
Mathematics and Latin	.40	.31
Mathematics and history	.33	.26
Mathematics and science	.41	.07
Algebra and geometry	.45	—

Thus, taking mathematics and science $r = .41$ (Burris) for an illustration, the meaning is if we take any group of mathematics students who rank x above or below the average, this group will on the average rank $.41x$ above or below the average in science. Suppose the class average in both subjects is 70 per cent., then the boy who obtained 100 in arithmetic might expect 82 per cent. in science.

The authors of these studies do not put any great stress on the exactness of their results. They seem to believe them to be but indications of correlations that may or may not exist. In this connection, Whipple says: "As a matter of fact, some of these published statements of correlation are actually wrong; correlations do not exist where they have been affirmed or do exist where they have been denied."

Burris says: "So far as the actual results are concerned (referring to his own results as given above), everything depends upon the reliability of the data with which the method deals; and . . . there are numerous reasons for questioning the value of the data so far used."

OBSERVATIONS ON PSYCHOLOGICAL TESTIMONY.

Now it seems to me that these conclusions can be drawn from the recent studies of the quantitative psychologists in regard to the correlation of mathematics and other subjects in the curriculum:

1. The psychologists are not in agreement among themselves on this question.

2. The conclusions are drawn from insufficient data. In physical science, in astronomy for instance, where fixed laws are postulated, and personality, except of the observer, does not

* Columbia University, Contributions, Vol. XI., No. 2.

enter the problem, often thousands of observations are used as a basis of a single result; in sciences, like psychology, where any result may be a function of so many phases of personality and environment, the basic data should be correspondingly increased. In the studies quoted above, the number of observations is far too small.

3. Some of the psychologists are expecting too high a value for the coefficient of correlation. Few practical teachers would say that because a boy did excellent work in algebra that he would do excellent work in Latin; but rather that in all probability he would do better work in Latin than a boy who failed in algebra.

Before leaving this topic which may seem to some an unwarranted digression from my main title, but which I regard as vitally connected with the whole curriculum and especially with mathematics, unless we become teachers of applied mathematics only, let me suggest that we enter the field of experimental psychology and personally investigate questions that bear upon our work.

It is tragic to think of investigators using the method of least squares, to which so many eminent mathematicians have contributed, to question the value of mathematical teaching for the general purposes of life.

SOME RESULTS OF THIS CRITICISM.

What will be the results of the present educational agitations and criticisms? Perhaps they may be summed up in this one word *correlation* better than any other word.

1. There must be a careful and intelligent correlation within the mathematical field. We now admit that there are no fixed boundaries between the divisions of mathematics. Text-books are appearing welding together arithmetic, algebra and geometry. This idea is only in its incipency and needs careful development. We need to encourage the correlative temperament. Isolated facts are difficult to retain and cannot be applied until they are correlated. In examining candidates for teachers' positions in some of the higher schools of Philadelphia, I have been astonished at the evident neglect of summarization and correlation on the part of young teachers.

2. There must be a discriminating correlation with the fields of applied mathematics. Too few of our teachers have at command even one field of application, such as physics, statistics, actuarial science, architecture, or engineering. One of the chief benefits of the modern movement is that many teachers are now studying the applications of elementary principles.

3. There must be a broad correlation with the non-mathematical studies such as the languages and history.

All good mathematics will translate into good English. If Latin and Greek are occupying the attention of the pupils, the etymology of the words used reveals their meaning. I have always found a class of Greek students to be interested in the propositions of Euclid in the original Greek. The history of mathematics will find a still larger place in our class rooms. It greatly adds the element of human interest.

4. There is an esthetic and ethical value of mathematics that is yet to be developed and appreciated.

The teacher who is not awake to the beauties of the science can never do first class work. The simplicity and the completeness of a proof or of a construction, the wonderful properties of a curve, or of a series, should awaken the enthusiasm of both teacher and pupils. For this reason, the early introduction of some more advanced topics is advisable even with secondary classes. A determinant solution of simultaneous linear equations in two or three variables awakens a lively interest; the geometry of Feuerbach's circle gives beautiful exercises on Books I., II. and III.; the fundamental properties of the parabola, ellipse and the cycloid and their applications will enthruse many a high school boy to press on to the Better Land of Higher Mathematics.

Character building is properly considered to be the chief work of the modern teacher. Mathematics is not wanting in its ethical bearing. "God is continually geometrizing," said Plato, expressing his idea of the dignity and beauty of the reason. The care with which we distinguish right from wrong in method and result, the habit of self-scrutiny in testing work, the element of fairness and equity exemplified in the equation,—all these and many other phases of our subject furnish fruitful ethical suggestions. As a climax, perhaps, we might place the

reverence for truth which mathematics should develop. The final appeal is never to personal opinion, nor to the authority even of the wisest, but to the simple truth as judged by the reason.

"As pure truth is to the pole star of our science," says Simon,* "so it is the great advantage of our science over others that it awakens more easily the love of truth in our pupils."

Another German writer has well put our claim—"Who does not know mathematics, and the results of recent scientific investigation, dies without knowing *truth*."

CONCLUSIONS.

However crowded the curriculum may be, mathematics will continue to occupy an important part of the educational program. The verdict of centuries of pedagogical progress cannot be reversed by the loud acclaims of a few radical agitators.

The mathematical teaching of the future must put still more stress on power of observation, reasoning, and invention. Our material must be rearranged with a view to presenting the vital parts as soon as possible. We are so accustomed to our traditional order that few of us conceive that other orders of presentation are possible. How little preliminary mathematics, for instance, is requisite to the solution of right and oblique triangles! Should logarithmic computation be confined to trigonometry?

Our courses in the future must also give more attention to the instruments of our craft. Why should we neglect the slide-rule, the planimeter, the transit and other forms of measuring instruments?

There must be an increasing confidence and respect among teachers, and among groups of teachers. The examination system is one of the obstacles which to a large extent interfere with harmony and mutual helpfulness. A brand on a piece of hardware, a guarantee of a reputable commercial house, the opinion of a learned jurist, carry weight and command respect. Why should not the opinion and the certificate of a teacher or of a school have meaning and force? The examination will always have a place but its place has been and is greatly magni-

* "Mathematical Unterricht."

fied, the *testimony* of the trained teacher is far more important. The future will magnify the *opportunity* and the *dignity* of the teacher. There will be more liberty of action, hence more responsibility. The conditions under which we work are becoming more and more diverse, hence no single method or text can be universal or general. Each must face his problems with discrimination, and each earnest worker should have the respect of all.

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In this world of imperfections we gladly welcome even partial intimacies. And if we find but one to whom we can speak out our heart freely, with whom we can walk in love and simplicity without dissimulation, we have no ground of quarrel with the world or God.

When we have fallen through story after story of our vanity and aspirations, and sit ruefully among the ruins, then it is that we begin to measure the stature of our friends; how they stand between us and our contempt, believing in our best.—*Robert Louis Stevenson.*